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Report on the PhD thesis of Mr. Arthur Vereijken

The thesis of Mr. Arthur Vereijken, entitled *Phenomenology of Glueballs in Effective Hadronic Models*, was prepared under supervision of prof. dr. hab. Francesco Giacosa and dr. Leonardo Tinti. The thesis is based on 3 research articles published in international peer-reviewed journals (Physical Review D), and 3 workshop proceedings.

This thesis utilizes effective models with hadronic degrees of freedom, guided by QCD global symmetries (notably chiral and dilatation symmetry) and their breaking. The main contributions are divided into four key areas:

1. Particle production in a scalar field theory: Studied a free scalar field coupled to a time-independent classical source as a simplified model for dilaton production in a medium. Demonstrated that the time evolution is highly non-trivial, contrary to historical expectations. Uncovered non-perturbative particle production similar to the Schwinger mechanism, featuring non-exponential decay and the quantum Zeno effect.

2. Analysis of tensor mesons: Applied the extended Linear Sigma Model (eLSM) to fit the decays of standard tensor mesons, confirming their quark-antiquark assignment. Used chiral symmetry to derive testable predictions for the masses and decay widths of axial tensor mesons, predicting them to be very broad states.

3. Identifying the tensor glueball candidate: Calculated the decay ratios of the tensor glueball using the eLSM and compared them with experimental data. Identified $f_2(1950)$ as the most likely tensor glueball candidate within this framework. Estimated that the tensor glueball could also be a relatively broad state.

4. Resolving the "13% rule" violation in charmonium decays: Investigated the decays of charmonium states (J/ψ and $\psi(3686)$) into vector–pseudoscalar final states, which significantly violate the perturbative QCD "13% rule" (most prominently in the $\rho\pi$ channel). Successfully obtained a phenomenological description of the observed $B(\psi(3686) \to h)/B(J/\psi \to h)$ ratios by incorporating electromagnetic effects and mixing with the vector glueball into the eLSM.

The thesis consists of 7 chapters, closed with 2 appendices and a rich bibliography (206 entries).

Chapter 1 is a concise introduction to Quantum Chromodynamics (QCD) and bound states in the theory (hadrons). It is followed by an overview of this thesis and main objectives together with the full list of publications based on which the thesis was written.

Chapter 2 is devoted to the analyses of vacuum decay in a simple scalar field theory in the presence of a static classical source. The exact scattering amplitudes and decay probabilities into n particles are derived, focusing on short- and long-time behaviors under specific external sources in a large volume. These findings are compared with previous work and the author discusses their phenomenological applications. It is emphasized that this study offers highly original theoretical insights: it demonstrates that conventional in/out-state formalisms and adiabatic switching-off methods fail because the interaction prevents the Hamiltonian and momentum operators from sharing eigenstates. The author and his collaborators elegantly resolve this by showing that the infinite-time limit must be taken after calculating time-dependent quantities. This is a novel approach using t states that could be valuable for other formal theories. Additionally, the author warns that standard shorthand volume approximations may introduce mathematical ambiguities in numerical prefactors.

Chapter 3 is devoted to a brief introduction to the extended linear sigma model (eLSM) for mesons and gluons, implementing spontaneous and explicit breaking of both chiral and scale symmetries in QCD.

Chapter 4 is devoted to the analyses of the lightest tensor mesons and their chiral partners, the axial tensor mesons, using eLSM introduced in the previous chapter. Based on the conventional quark-antiquark assumption, the predictions for the masses, strong decays and radiative decays of ground-state tensor mesons align well with empirical data, confirming their PDG assignments. For the axial tensors, the masses are shifted slightly higher due to the chiral condensate. Although subject to large uncertainties, these states are shown to be broad, with their dominant decay channel being a vector-pseudoscalar pair. Whereas their broad nature makes experimental detection challenging, specific promising channels have been identified for discovering states. The author states that the tensor glueball (predicted around 2.2 GeV) can be directly implemented in the eLSM to resolve current experimental mixing ambiguities.

Chapter 5 is devoted to the study of the tensor glueball by constructing chirally invariant Lagrangians to determine its decay ratios. The analysis reveals that the dominant decay channels involve two vector mesons, $\rho\rho$ and $K^*\bar{K}^*$. Given the estimated $\pi\pi$ mode, the large $\rho\rho/\pi\pi$ ratio is predicted and this suggests that the tensor glueball is very broad.

Chapter 6 addresses the $\rho\pi$ puzzle, which is the breaking of the 13% rule in charmonium decays, by mixing $\psi(3686)$ with a vector glueball and incorporating electromagnetic decays via vector meson dominance. Using three free parameters, the model demonstrates that while electromagnetic interactions are significant, vector glueball mixing is essential to accurately reproduce the experimental vector-pseudoscalar data. In order to refine these findings and confirm the mixing, the model should be expanded such that it describes J/ψ and $\psi(3686)$ decays separately rather than just their ratio. This extension could constrain the mixing angle and provide crucial decay estimates to guide the experimental discovery of the vector glueball.

Chapter 7 is devoted to a brief summary of this thesis and future research plans.

The thesis is well motivated and written in a pedagogical manner. Every chapter is closed with a short summary, and this is helpful to keep track of involved formalism.

There arise the following remarks and questions:

1. Whereas Chapter 2 focuses on a formal framework of vacuum decay under an external classical source, the subsequent chapters utilize the standard eLSM approach for phenomenology. How exactly do these two parts connect conceptually? Is there a way to rigorously map the classical source dynamic from Chapter 2 onto the explicit decay processes within the eLSM?

2. In Chapter 4, Eq. (4.2.7) in Section 4.2: clarify why G_0 scales as $\mathcal{O}(N_c)$ rather than $\mathcal{O}(N_c^2)$? In standard large-N_c QCD, the vacuum energy density and gluon condensate typically scale as $\mathcal{O}(N_c^2)$ due to the color degrees of freedom.

3. In Chapter 6, an effective Lagrangian is applied to heavy and light hadron systems. Explain what specific role Heavy Quark Symmetry plays in this framework, how it interplays with the light-quark chiral symmetry, and how it constrains the model parameters/dynamics.

4. In Chapter 6, the author mentioned the inclusion of electromagnetic decay channels through vector meson dominance (VMD). It was not entirely clear to me where exactly VMD was applied in the concrete formulas or calculations.

Conclusions

The thesis of Mr. Arthur Vereijken is compelling and contains new and original results on the glueball phenomenology. Further extensions of the current framework are of particular interest, in view of recent progress in QCD experiments.

I conclude that the thesis clearly complies with all the formal and customary requirements. I would like to warmly recommend this thesis for a public defense.

C. Sasaki

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