dr hab. Mariola Kłusek-Gawenda, prof. IFJ PAN
Zakład Teorii Silnych Oddziaływań i Układów Wielu Ciał
Oddział Fizyki Jądrowej i Oddziaływań Silnych
Instytut Fizyki Jądrowej im. Henryka Niewodniczańskiego
Polskiej Akademii Nauk
ul. Radzikowskiego 152, 31-342 Kraków

Kraków, 5 June 2026

Review of the Doctoral Dissertation
by Arthur Vereijken,
entitled *Phenomenology of Glueballs in Effective Hadronic Models*

The dissertation was prepared at Jan Kochanowski University, Doctoral School, Faculty of Natural Sciences, Physical Sciences, under the supervision of Professor Francesco Giacosa and Dr Leonardo Tinti.

The dissertation concerns the phenomenology of glueballs and selected hadronic states in effective hadronic models. Glueballs, understood as hypothetical bound states composed solely of gluons, constitute one of the most challenging problems in QCD. Their unambiguous experimental identification still remains unattained, which underlines the importance of theoretical studies and phenomenological models capable of linking the symmetries of QCD with experimental results. In this respect, the dissertation by Arthur Vereijken addresses a topical and important issue in contemporary hadron physics. QCD in the low-energy regime is a strongly coupled theory; therefore, standard perturbative methods are insufficient in this domain. The Author, therefore, rightly adopts a model-based approach, in which the basic degrees of freedom are hadrons, while the construction of the model is guided by the global symmetries of QCD and their anomalous, explicit and spontaneous breaking patterns. Chiral symmetry and dilatation symmetry play a particularly important role in the dissertation. This choice is well justified from the physical point of view. These symmetries are not merely a formal addition to the description, but they genuinely organize the structure of the models used in the subsequent chapters. As a result, the dissertation has a clear point of reference: it is not an arbitrary fitting of data, but rather an attempt to provide a phenomenological description of hadrons consistent with the basic properties of QCD in the low-energy regime.

1. Structure of the dissertation

The dissertation has a clear structure. It consists of seven chapters, two appendices and a bibliography. It also contains a list of 13 figures and 26 tables. The first chapter serves as a theoretical introduction. The second chapter concerns a scalar glueball with a time-independent classical source. The third chapter introduces the extended Linear Sigma Model, further denoted as eLSM. The fourth chapter discusses spin-2 tensor mesons and axial tensor mesons. The fifth chapter is devoted to the tensor glueball. The sixth chapter concerns the vector glueball, mixing and the $\rho\pi$ puzzle. The seventh chapter contains the summary.

The dissertation is based on several scientific publications. Chapter 2 is based on material published in 2024 in *Phys. Rev.* ***D*** [13], Chapter 4 on work published two years earlier in the same journal [14], Chapter 5 on material published in 2023 in *Phys. Rev.* ***D**** and on two proceedings [15], [16] and [17], while Chapter 6 was presented in a compact form in a proceedings contribution from 2025 [18]. This is important, as it shows that a significant part of the results is already present in scientific circulation. At the same time, the dissertation does not give the impression of being a simple compilation of articles. The Author provides a common introduction, a common motivation and consistently guides the reader through the topic of glueballs in effective hadronic models.

In my opinion, however, it would be worthwhile for the doctoral candidate to clearly indicate during the defence his individual contribution to the respective parts of the dissertation, especially since several chapters are based on co-authored publications. This concerns, in particular, analytical derivations, numerical calculations, fits to data and the physical interpretation of the results. This should not be treated as a criticism, but rather as a natural review question in the case of a dissertation based on several scientific papers.

2. Content of the individual chapters

Chapter 1 introduces the reader to the basic elements of QCD and hadron physics. The Author discusses asymptotic freedom, confinement and the problem of bound states in QCD. It is important to recall that experimentally we observe hadrons, not free quarks and gluons. This naturally justifies the use of effective models in which the degrees of freedom are hadronic objects. In the same chapter, the Author presents the classification of mesons. Table 1.1 is useful, as it systematizes conventional $q\bar{q}$ mesons up to spin $J = 2$. This provides good preparation for the subsequent chapters, where tensor mesons, axial tensor mesons and candidates for unconventional states, including glueballs, appear.

Chapter 2 is one of the most formal parts of the dissertation. The Author analyzes a free scalar field coupled to a time-independent classical source. This model is treated as a simplified description of dilaton production in a medium. In the language of the dissertation, the dilaton is related to the scalar glueball. The starting point is the action of the model and the equation of motion for a scalar field with a classical source. This is one of the most original and theoretically interesting parts of the dissertation. The Author shows that the theory with a static source is not trivial, contrary to earlier intuition. In particular, he indicates that non-trivial dynamics appears through particle production from the vacuum, conceptually similar to the Schwinger mechanism. This demonstrates that the doctoral candidate does not limit himself to a mechanical application of known arguments, but is able to critically analyze formal assumptions. An important result of this chapter is the description of vacuum decay. The Author shows that the probability of transition from the vacuum to many-particle states takes the form of a Poisson distribution. He also discusses non-exponential decay, saturation at large times and the quantum Zeno effect. It is particularly interesting that the Author clearly distinguishes his result from the classical intuition associated with the Schwinger mechanism.

Chapter 3 introduces the extended Linear Sigma Model. This is the central tool used in the subsequent part of the dissertation. The Author discusses the dilaton potential, spontaneous chiral symmetry breaking and the three-flavor Linear Sigma Model. This chapter is necessary, because without it the later applications to tensor mesons, the tensor glueball and the vector glueball would be much more difficult to follow. An important advantage of this part is, in my view, the consistent use of the language of symmetries. The Author does not treat eLSM as a purely technical tool for fitting data. The model is constructed so as to reflect the most important properties of QCD in the low-energy range: chiral structure, trace anomaly, dilatation symmetry breaking and the appropriate assignments of mesonic multiplets.

Chapter 4 concerns spin-2 tensor mesons and axial tensor mesons. The Author extends eLSM to include the spin-2 sector. This is a natural step, since tensor mesons and axial tensor mesons are chiral partners. This makes it possible to use data for well-known tensor mesons to predict the properties of less well-known axial tensor mesons. The Author fits the model parameters to known experimental data and then obtains predictions for the masses and decays of axial tensor mesons. The conclusion that these states are broad states is physically interesting, since it may explain the difficulties in their experimental identification. It should be kept in mind, however, that the Author himself points to large uncertainties and to the dependence of some predictions on the parameters of eLSM.

Chapter 5 is devoted to the tensor glueball. The Author constructs a chiral model describing tensor glueball decays, then calculates decay ratios and compares them with data for known experimental candidates. It is important that the analysis is not based solely on the mass of the resonance, but also takes into account decay patterns and comparison with available data. One of the most important results of the dissertation is the indication of $f_2(1950)$ as the best candidate for the tensor glueball within eLSM. This is a clearly formulated phenomenological result that may be useful for further experimental analyses. At the same time, it requires caution. This conclusion depends on the adopted model, the quality of the available data and the assumptions concerning possible mixing with conventional $q\bar{q}$ states. In my opinion, the formulation “the best candidate within eLSM” is more appropriate than a stronger claim of an unambiguous identification of the tensor glueball. The Author himself notes that lattice QCD usually indicates the tensor glueball mass range of about 2.2–2.4 GeV, which may make $f_2(1950)$ too light a state. On the other hand, the absence of obvious contradictions with the available data and the width of this resonance make it an interesting candidate. It is particularly valuable that the Author does not conceal the limitations of this interpretation. He indicates that further data are needed on tensor meson decays into vector-vector, pseudoscalar-pseudoscalar, tensor-pseudoscalar and nucleon-antinucleon channels. He also emphasizes that the analysis does not include mixing of the tensor glueball with conventional meson states, which may be important especially for excited states with similar masses.

Chapter 6 concerns the vector glueball, mixing and the $\rho\pi$ puzzle. The Author analyzes the decays of J/ψ and $\psi(3686)$ into vector-pseudoscalar channels, denoted in the dissertation as VP channels. The starting point is the perturbative QCD “13% rule”, which is violated in many channels, particularly in the $\rho\pi$ channel. The Author shows that including electromagnetic decay channels and mixing with the vector glueball improves the description of the data. This is an interesting result and fits well into the subject of the dissertation. However, the improvement of the fit should not be treated as unambiguous evidence for the existence of such mixing. The Author himself writes cautiously that, at present, mixing of $\psi(3686)$ gives slightly better agreement with data, but this is probably not yet sufficient to state definitively that the glueball mixes with $\psi(3686)$. I consider it important that Chapter 6 does not present the result as a final solution to the $\rho\pi$ puzzle. The Author indicates that a more complete analysis should include higher-order strong interaction diagrams, possible different interaction strengths for mesons with a strange quark, more decay channels and a separate treatment of J/ψ and $\psi(3686)$ decays. In this sense, Chapter 6 constitutes a valuable model analysis, but it does not close the physical problem.

Chapter 7 synthetically summarizes the most important results of the dissertation. The Author returns to four main topics: the non-trivial dynamics of a scalar field with a static source, predictions for axial tensor mesons, the interpretation of $f_2(1950)$ as a tensor glueball candidate and the description of the violation of the 13% rule in charmonium decays.

3. Strengths of the dissertation

The first strength of the dissertation is the well-chosen topic. Glueballs are predicted by QCD, but they have not been unambiguously identified experimentally. The Author therefore addresses a genuinely important problem. This is not a closed or merely technical topic.

The second strength is the consistent use of symmetries. The Author does not build the model arbitrarily. The point of reference is provided by the global symmetries of QCD and their anomalous, explicit and spontaneous breaking patterns. This organizes the whole dissertation well and gives it coherence.

The third advantage is the combination of formalism with phenomenology. Chapter 2 has a more formal character and demonstrates the Author's proficiency in working with quantum field theory. Chapters 4, 5 and 6 are closer to phenomenological analysis and comparison with data. This shows that the doctoral candidate is able to work both at the formal level and at the model-phenomenological level.

The fourth strength is the concreteness of the results. The Author does not stop at general considerations. He provides masses, decay widths, branching ratios, decay ratios, fit parameters and comparisons with data. This allows the reader to assess where the model works well and where caution is required.

The fifth advantage is that the Author formulates predictions that can be further verified. This concerns, in particular, broad axial tensor mesons, tensor glueball decay channels and the influence of vector glueball mixing on VP decays of charmonium states. This is important because the dissertation may serve as a point of reference for further experimental and phenomenological analyses.

I also positively assess the fact that part of the results has already been published or prepared in the form of scientific publications. The dissertation is built on six scientific publications, including three papers published in *Physical Review D*. Some of these works have already attracted citations, which indicates that the results have entered the scientific discussion in the field. Moreover, the presence of conference proceedings and presentations shows that the doctoral candidate has actively presented his results to the international hadron-physics community.

4. Critical remarks and weaker aspects of the dissertation

The most important critical remark concerns the coherence of the narrative. The dissertation is thematically coherent, since all its main parts concern glueballs, effective hadronic models and eLSM. In some places, however, one can see that the individual chapters originate from separate publications. The transition between the formal Chapter 2 and the more phenomenological Chapters 4-6 could have been stronger. Brief comments linking these parts would help the reader better grasp the overall logic of the dissertation.

The second remark concerns the presentation of model uncertainties. The Author discusses them in many places, but in my opinion it would be useful to collect them in a more systematic way. The reader would like to know clearly which predictions are stable and which depend strongly on eLSM parameters, the choice of decay channels, assumptions about mixing or the quality of the experimental data.

The third remark concerns the interpretation of $f_2(1950)$. This is one of the most interesting results of the dissertation, but also one that requires considerable caution. The Author shows that $f_2(1950)$ is the best candidate within the considered model and the available data. However, this is not an unambiguous identification of the tensor glueball. Particularly important are possible mixing effects with conventional meson states and the fact that lattice QCD usually indicates a higher tensor glueball mass range.

The fourth remark concerns Chapter 6. The model with electromagnetic effects and vector glueball mixing improves the description of the data for VP channels. This is important. However, an improvement of the fit is not yet proof of a physical mechanism. The Author himself indicates that further steps are needed: more channels, higher-order strong interaction diagrams, possibly different couplings for mesons containing a strange quark and a separate treatment of J/ψ and $\psi(3686)$ decays. In my opinion, this is a properly formulated limitation, which is also worth emphasizing in the review.

The fifth remark is editorial in character. The dissertation contains many tables and formulae, which is understandable given the subject. At times, however, short comments after the most important tables would be useful. Ideally, such comments would explain in simple terms what is an input to the model, what is a result and what is only an auxiliary coefficient. This is not a substantive flaw, but rather a matter of reading comfort.

5. Presentation of the results

The presentation of the results is generally good. The dissertation contains an appropriate number of tables and figures. The figures in Chapter 2 clearly illustrate time dependence of particle production and vacuum survival probability. Thanks to them, the formal part of the work is easier to understand.

In Chapter 5, the summaries concerning tensor glueball candidates are useful. Particularly important are Table 5.4, where decay ratios are compared with available data, and Table 5.6, where the status of individual isoscalar tensor resonances as tensor glueball candidates is presented.

In Chapter 6, Figure 6.3 is useful, as it shows the results of fits with and without mixing with the vector glueball. The figure clearly illustrates that adding mixing improves the description of part of the data, but it also shows that the problem is not completely closed.

6. Questions to the doctoral candidate

Several of the questions below are intended to clarify points that were not fully clear to me while reading the dissertation. At the same time, a substantial part of them may be treated more broadly as an invitation to discussion, since they concern the interpretation, possible extensions and further development of the results presented in the dissertation.

1. Since a significant part of the dissertation is based on published work or work prepared in co-authorship, I would like to ask the doctoral candidate to briefly indicate his individual contribution to the respective parts of the dissertation: analytical derivations, numerical calculations, fits to data and the physical interpretation of the results.

2. In Chapter 2, the Author assumes a time-independent classical source. What are the physical limits of this approximation? In the context of the medium created after heavy-ion collisions, can one indicate situations in which this assumption is particularly good?

3. Would the introduction of a slowly time-dependent source preserve the main features of the result, namely non-trivial time evolution and particle production from the vacuum, or is the result strongly connected with the idealization of a static source?

4. In Chapter 4, the Author predicts that axial tensor mesons are broad states. Is the width of these states the main reason for the difficulty of their experimental observation? Which decay channels would, in the Author's opinion, be the most promising?

5. The Author indicates $f_2(1950)$ as the best candidate for the tensor glueball within eLSM. How strong is this conclusion after taking into account possible mixing with conventional $q\bar{q}$ states?

6. Which decay channels would be the most decisive for confirming or rejecting the interpretation of $f_2(1950)$ as a tensor glueball?

7. In Chapter 6, the description is improved by the combination of electromagnetic decay channels and mixing with the vector glueball. Which of these components is, in the Author's opinion, physically more important for understanding the $\rho\pi$ puzzle?

8. Are there alternative mechanisms that could provide a similar description of the breaking of the 13% rule without invoking vector glueball mixing?

9. Which predictions of the dissertation are, in the Author's opinion, most robust against changes in the details of the model, and which depend most strongly on the specific form of the eLSM Lagrangian?

10. In Fig. 6.3 the impact of vector glueball mixing appears to be channel-dependent. In most channels the result with mixing is larger than the result without mixing, while for the $\rho\pi$ channel, and apparently also for $\omega\eta'$, the inclusion of mixing lowers the predicted value. Could the author explain whether this behaviour follows mainly from destructive interference between the J/ψ and effective glueball contributions, or from the channel-dependent flavor coefficients κ_i?

11. In Eq. (6.2.19) the author introduces a form factor with an energy scale Λ and states that this scale will be fitted. In Table 6.2 the fitted value $\Lambda = 1.6 \cdot 10^3$ MeV is given. Could the author clarify how this value was obtained in practice? In particular, what was the fitted objective function, were experimental uncertainties used as weights, and how stable is Λ with respect to the choice of fitted channels?

12. Does the Author see a possibility of a more direct comparison of the results with lattice QCD, not only through masses, but also through amplitudes, ratios or other observables?

7. Final remarks

The doctoral dissertation by Arthur Vereijken is a valuable work in the field of the phenomenology of QCD in the low-energy regime. The Author has a good command of the tools of effective hadronic models and is able to use them both formally and in connection with experimental data. The dissertation demonstrates a good knowledge of eLSM, QCD symmetries, the problem of glueballs and meson phenomenology. I consider the most important results of the dissertation to be the analysis of the non-trivial dynamics of a scalar field with a time-independent classical source, predictions for axial tensor mesons, the indication of $f_2(1950)$ as a particularly interesting candidate for the tensor glueball within eLSM and a model description of the breaking of the 13% rule in charmonium decays, including electromagnetic effects and vector glueball mixing.

My critical remarks mainly concern the scope of interpretation of the results, the presentation of model uncertainties and the need for caution in the physical interpretation of $f_2(1950)$ and vector glueball mixing. They do not undermine the value of the dissertation. On the contrary, they indicate places where the work could be even more convincing and useful for further analyses. Irrespective of the comments formulated above, I would like to emphasize that the dissertation also has considerable pedagogical value and may serve as an excellent introduction to the subject for young researchers entering the field.

I conclude that the dissertation meets the requirements imposed on doctoral dissertations in the discipline of physical sciences. It contains original scientific results, is well embedded in the literature and shows that the doctoral candidate is able to conduct theoretical and phenomenological research independently.

One of the most valuable aspects of the doctoral dissertation by Arthur Vereijken is the skilful combination of advanced theoretical formalism with the phenomenological analysis of hadrons. The Author does not limit himself to purely formal considerations, but consistently carries out analyses that remain directly connected with experimental observations. In this way, the dissertation constitutes an important contribution to the discussion on the identification of elusive hadronic states, such as glueballs. Particularly noteworthy is the original analysis of the dynamics of a scalar field with a static source, revealing non-obvious phenomena related to particle production from the vacuum, as well as the consistent use of the extended Linear Sigma Model for the systematic description of tensor mesons and for formulating predictions concerning the tensor glueball candidate, $f_2(1950)$. The dissertation not only enriches the existing literature with new predictions and well-justified interpretations, but also indicates directions for further research by offering concrete and testable phenomenological predictions. Taking into account the scientific quality of the dissertation, the relevance of the topic, the combination of formal analysis with phenomenological applications, the concrete predictions obtained within effective hadronic models, and the fact that the dissertation is based on a publication record that includes, among others, papers published in *Physical Review D*, in my opinion the dissertation goes beyond the standard requirements for doctoral theses. Therefore, I recommend that the doctoral dissertation be awarded distinction.

I therefore recommend that Arthur Vereijken be admitted to the further stages of the doctoral proceedings, including the public defence of the doctoral dissertation.