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# Report on the doctoral thesis

# „Phenomenology of Glueballs in Effective Hadronic Models"

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### Introduction

The presented dissertation discusses the phenomenology of glueballs. The latter are massive bound states of gluons, present and expected in many theoretical approaches to Quantum Chromodynamics (QCD). Yet, despite many years of efforts on both the theoretical and experimental sides, no decisive experimental confirmation has been reached. The problem provides a very nice example of theory – experiment interaction, since any experimental signal of a bound state has to be put in a consistent theory framework in order to correctly interpret the constituent structure of that state. The subject is hence very interesting and active, as new significant experimental results are arriving (for example, Phys. Rev. Lett. **132**, 181901 by the BESIII collaboration) and ask for a theoretical discussion. The Author approaches glueballs from several points of view, providing new, significant theoretical insight. In the first part of the dissertation, the Author re-analyses the problem of particle production in a generic quantum field theory in a non-trivial classical background, reaching interesting conclusions. In the second part, the Author introduces, extends, and applies the extended Linear Sigma Model to interpret the existing experimental data and to provide new predictions concerning the glueball spectrum and decays.

I believe that the thesis provides substantial, important, and timely new results in the field, which I will try to argue for with the subsequent discussion.

### Thesis structure

The thesis consists of seven chapters. The first chapter provides a brief and gentle introduction to the topic of Quantum Chromodynamics (QCD) with an emphasis on the bound state spectrum of the theory. The following chapters: 2, 4, 5, and 6 describe the particular achievements of the Author based on his publications. Chapter 3 is used to introduce the reader to the topic of Linear Sigma Models, presenting its motivations and applications in phenomenology of QCD bound states. Chapter 7 contains a short summary of the obtained results and an outlook providing possible future research directions. The thesis is written very well, with a concise but precise language. The Author guides the Reader through the logic of the derivations. He is asking questions and then provides answers, explaining the reasons for

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consecutive steps. I like this way of writing the narrative as it involves the Reader and allows him to go through sophisticated steps without unnecessary weight. Figures are of very good quality, with large labels, clear captions, and illustrate nicely and precisely the discussed matters

## Contents

The discussion presented in Chapters 2, 4, 5, and 6 clearly demonstrates the Author's ability to perform innovative, original theoretical research. Particularly, I highly value his critical assessment of existing, accepted by the community, conclusions described in Chapter 2.

### The scalar glueball with a time-independent classical source:

Chapter 2 is based on a publication in Phys. Rev. D 110 (2024) 11, 116004 and summarizes new findings concerning the interaction of a scalar quantum field theory in 4 space-time dimensions with a classical background. Although this problem counts nowadays as a classic problem discussed in many quantum field theory textbooks, the Author sheds new light and obtains significant new results. I appreciate the section where he compares his approach with the material from Coleman's lecture, exhibiting a deep understanding of quantum field theory.

Although it was partially argued that the motivations for this part of the work were connected with possible glueball production in a medium, this was not sufficiently commented on in the conclusions. I am missing some sentences linking the new theoretical results with possible phenomenological signatures.

**The extended Linear Sigma Model:**

The Linear Sigma Model is an effective theory of QCD at low energies. In contrast to Chiral Perturbation Theory, it keeps both scalar (sigma) and pseudoscalar (pi) degrees of freedom. Based on the analysis of symmetry breaking patterns, the Author extends the Lagrangian describing these two degrees of freedom by the interaction with the effective gluon field G. The discussion of the Lagrangian for the G field is based on the dilatation symmetry breaking. Chapter 3 contains the motivation and exposition of the Linear Sigma Model with 3 quark flavors.

Chapters 4 and 5 contain original new results obtained by extending the previously described Lagrangian to include spin-2 tensor and axial-tensor mesons (Chapter 4) and the tensor glueball (Chapter 5). The extended Linear Sigma Model allows the Author to derive formulae for the masses of tensor and axial tensor mesons and subsequently their decays. A similar strategy is applied in Chapter 5 to the tensor glueball. Table 5.2 shows an example of such predictions that can now be contrasted with experimental results. Such results not only enable the Author to interpret existing experimental data, but also provide hints for subsequent directions of experimental searches. The discussion contained at the end of Chapter 5 demonstrates clearly the Author's broad knowledge and understanding of the theoretical model and experimental data, which I appreciate a lot.

What I am missing, however, in the presentation of the results, is a clearer distinction of the input to the theoretical framework and its output. The free parameters of the Lagrangian, the couplings h_i and g_i, have to be fixed by some of the experimental data. Once fixed, the remaining quantities are the model's predictions. It would be very helpful to distinguish more clearly the experimental results that were taken as input.

**The vector glueball, mixing, and the ρπ puzzle**

In Chapter 6, the Author describes the application of the developed framework to the problem known as the ρπ puzzle. I believe this is a very nice extension of the results obtained earlier with the extended Linear Sigma Model. The Author convincingly presents the experimental data and explains the current status. He then introduces mixing with a vector glueball and derives predictions for the quantity Q_h. The inclusion of the mixing with the vector glueball allows him to describe the experimental data much better.

I must stress that the thesis touches on some very important subjects, and it presents significant results. The Author convincingly demonstrates his ability to perform original work and to deal with complicated theoretical output.

## Remarks and questions

Reading the thesis, I had the following remarks and came across some general questions:

1) I was missing a broader discussion of lattice QCD results concerning the glueball spectrum. For example, in the opening of Chapter 5, the Author cites Ref 148 as the only unquenched calculation of the mass of the tensor glueball, which is from 2012. I understand that the glueball spectrum is still a very challenging problem for lattice QCD, and the progress might be slow. Explaining that in a short paragraph would allow the Reader to better appreciate the quality of the results obtained and described in the thesis.

2) While reading the thesis, I found that it is not clear how the unknown constants g_2, g_3, ... g_6 and h_1, h_2, and h_3 in the Lagrangian 3.3.29 are fixed. Some of them seem to be renamed/extended in subsequent sections, for example, Eq. 4.2.2 contains h_1^ten, h_2^ten, and h_3^ten. As mentioned earlier, it would be helpful for the Reader to provide a short summary of the inputs used to fix these parameters.

3) Developing on the previous point, although I understand that that might be a big enterprise, I believe it would be beneficial for the results to be accompanied by some sort of uncertainties coming from the way the constants in the Lagrangian were fixed. Naively, I would expect that one could propagate the statistical uncertainties of the experimental results used to fix the coupling constants to the final predictions.

4) In Eq. 3.2.6, the mass of the sigma particle is given by – 2 m_0^2. Since m_0^2 is taken to be negative, m_sigma^2 is positive. How should I then understand Eq. 3.2.8 and 3.2.9, where the term m_0^2 enters without a minus sign, thus giving negative masses for pi and sigma?

However, the list of these minor remarks should not convey a wrong impression. The thesis is written with care, precise language, and clear figures. The suggested literature is vast and diverse. References show that the Author is fluent in the field of expertise.

## Conclusions

Overall, the thesis presents substantial advancements in the field. The Author has already published four papers in renowned international physics journals. He has presented his research at multiple international conferences and workshops, which is certified by several published proceedings. The thesis fulfills all the requirements presented by the law (art. 187 Ustawy z dnia 20 lipca 2018 r. Prawo o szkolnictwie wyższym i nauce (Dz. U. z 2018 r. poz. 1668 z późn. zm.)) and by custom, and I therefore ask to allow further proceeding of the Author's application.

Podsumowanie: przedstawiona rozprawa zawiera znaczące wyniki w dziedzinie obliczeń teoretycznych przewidywań dla masywnych stanów związanych gluonych zwanych glueballami. Autor opublikował cztery artykuły w renomowanych, międzynarodowych czasopismach. Przedstawiał swoje wyniki na wielu międzynarodowych konferencjach i warsztatach, co jest potwierdzone opublikowanymi artykułami z tych wystąpień. Stwierdzam zatem, że recenzowana rozprawa doktorska **Arthura Vereijkena** spełnia warunki określone w art. 187 Ustawy z dnia 20 lipca 2018 r. Prawo o szkolnictwie wyższym i nauce (Dz. U. z 2018 r. poz. 1668 z późn. zm.) i wnioskuję do Rady Naukowej Instytutu Fizyki Uniwersytetu Jana Kochanowskiego w Kielcach o dopuszczenie **Arthura Vereijkena** do dalszych etapów przewodu doktorskiego.

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